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## STUDIES IN THE STRUCTURE OF SYSTEMS

### 1. THE SEPARATION OF PROBLEMS

MATHEMATICS, physics, astronomy were mere chapters in the philosophy of Greece. Gradually they became conscious of their own problems and matured into relative independence. This process of emancipation is typical: many classes of problems (an infinite number perhaps!) are contained in the realm of what is commonly called philosophy; more or less vague efforts at solving them are made, and these absorb the attention for a while until there comes a day when one class is recognized as distinct from the others, and a new science is born out of philosophy. Mathematics arrived at this condition early; physics only in the days of Galileo. It is true the experimental method which he used would alone assure him immortality. But it is not this new method which emancipated physics; it is the particular type of problem which Galileo set. At first sight it might seem limited in application and insignificant in interest; but it proved fruitful in calling forth other problems of the same type, in whose solution the same or similar methods of procedure were effective. Singling out a new type of problem gave birth to a new science.

This process of emancipation of philosophy's progeny is going on vigorously even to-day. It seems only yesterday that chemistry was born; and now psychology is asserting with gentle emphasis that it is weaned from the mother milk of philosophy!

In the realm of the old Aristotelian logic there are four distinct classes of problems that are still treated promiscuously and without regard for their inherent distinctions. Solutions of one are given out for solutions of another, though in reality they may be perfectly irrelevant to it. To separate these disciplines by clearly distinguishing the kinds of problems which they present will greatly help in their development; it is the first and indispensable step toward their proper solution.

They all refer principally to what may be called "cognition," *i. e.*, knowledge of a certain kind characterized by the property of being coherent, necessary, systematic, etc. Mathematics, physics, may stand as examples of what is here designated by cognition and may always be substituted for it in the present discussion.

Suppose a system of cognition, such as plane geometry, to be given, *e. g.*, in the form of Euclid's "Elements." Various questions may be asked regarding it, such as: Are the propositions clear and convincing? Do you grasp them readily, or only with difficulty? Are the "axioms" more "evident" to you than the propositions? How is your attitude toward the truth of a proposition affected by the "proof" which is given of it? Is your study aided or impeded by "logical rigor" in the formulation of the "axioms" and the arrangement of the propositions? How were these propositions discovered, and what natural conditions are most favorable for discovering new ones? These questions can easily be multiplied indefinitely. They are all of a certain type which may be characterized as follows: They imply that, besides the system of geometry, an "I" or "you," in general, a "consciousness," a "subject" is given, and the questions concern the relation of this "consciousness" to the system of propositions of geometry. Both are, in the meaning of the questions, separate, distinct, though in relation to each other. What this relation is in particular is not stated. The propositions of geometry may be conceived as "acts" of this "subject," as "content" of this "consciousness," and thus as residing in this "ego"; but the words "acts," "content," "in" indicate again special relations of these propositions to the "subject," just as did "evidence," "clearness," "difficulty of apprehension." Any question even whether a proposition may "exist" independently of a "human consciousness" or whether it is, first and last, nothing but a "content of *some* human consciousness" must be considered to be of the same type.

In order to make this clearer let us call the propositions, concepts, etc., such as form plane geometry, *"logical entities,"* and let us say that they have "existence" in a definite realm which we will call the "realm of logical entities." Distinct from this "realm of logical entities" is the "ego" which enters into (or is in) relation to them; and I shall call this relation the *"subject-relation"* of the logical entities. Some such distinction is indeed required by any of the above questions and it does not in any way prejudice the decision as to *what* the subject-relation will be: the whole realm of logical entities may be "immanent" in the "ego," or "transcendent" —immanence and transcendence would merely express definite kinds of subject-relation.

It is apparent that what ordinarily passes for "idealism" is concerned primarily with problems regarding the subject-relation. And it has often passed for "obvious" that a consideration of the subject-relation is primary, indispensable, unavoidable, decisive. Quite on the contrary it seems to me necessary to recognize that problems regarding the subject-relation are merely *one* type of problems, that other, distinct types of problems are possible and important in the solution of which a decision regarding the subject-relation is *irrelevant.* This does not derogate in the least from the importance of carefully studying the subject-relation.

It seems to be recognized more and more that problems of this type belong to psychology; and I shall therefore speak of *"Psychology of Cognition"* to designate the discipline which studies the subject-relations of logical entities. It may be that this type of problems can fruitfully be subdivided; it may be that this whole type should be classified differently; it will have no bearing on the present study, so long as problems regarding the subject-relation of logical entities are recognized as distinct from others regarding the logical entities themselves.

It is a platitude that nothing is *true* for me unless it is true for *me*—though much discussion has hinged on this platitude. An extreme individualism has based on it the theory that no truth exists for me, unless it is recognized, seen, apprehended as such by me. All those who urge "evidence" as *the* test of logical truth, maintain in the last resort, or frankly even from the beginning, this theory. They base "logic" on "psychology"; for "evidence" is one kind of "subject-relation." So long as this subject-relation remains the problem under investigation, their claim may be made with much force. We seem indeed to be constantly guided in our search for truth by the "clair et évident" of Descartes—though it may well be suspected that the subject-relation corresponding to what we call "truth" is much more complicated, as the pragmatists are showing with convincing force. But when we set the problem of the truth of a proposition, *apart* from its power of convincing me or you, provided such a problem is admitted as possible, we enter a completely different realm of investigation. Still clearer is this when we state such problems as: Does proposition $p_1$ "imply" $p_2$, or not? What is the exact relation of $p_1$ to $p_2$? Can $p_2$ be "proved" by assuming $p_1$? We then do not ask: Can "we" prove $p_2$, but: Can it be proved? And these two do not by any means coincide. We must often admit the logical existence of relations, though "we" are unable to exhibit them. Every algebraical equation *has* a root, *i. e.*, a root "exists," but given an algebraical equation "we" can but rarely find it. It is by no means necessary to go to special cases in mathematics to show the distinction between "logical" and "psychological" existence, though it is good to give a radical example where no "knower" exists who psychologically "perceives" the existence of the logical entity. For whilst in the ordinary examples we readily admit that a certain relation may logically exist between $p_1$ and $p_2$, though "you" or "I" do not "see" it, *i. e.*, though it does not exist for "us," we are apt to overlook the radical nature of the distinction, because we may still, and often rightly, say: but it exists "psychologically" for "somebody." This blurs the distinction; for instead of entering into the purely logical question of the relation of $p_1$ to $p_2$, we fall back into the psychological question of the subject-relation of this logical relation, by reiterating: but the relation $R$ must be in subject-relation to "somebody," some "consciousness," some "subject," some "knower"! This is a mixing of problems, for the question was not: How do we, or how does anybody, perceive, or find, or in whichever manner establish a subject-relation to $R$; but: does it "exist"; does the proposition that "the sum of the angles in a plane triangle is equal to two right ones" presuppose the "parallel axiom"?

To some it will, no doubt, be quite impossible to "ignore," for the time being, the psychological problem of the subject-relation, and to them the "realm of logical entities" will always flit around some "consciousness"; as the platonic ideas always had physical existence somewhere, as Kant's "transcendental ego" was hidden in the innermost depths of the brain. And yet, it is just this *"ignoring"* of one problem when moving in the realm of another which is so characteristic of all fruitful work: in any "object" many kinds of problems intersect; properly and systematically to ignore the "others" is the first and necessary step toward the solution of the "one."

Whatever theory is accepted regarding the subject-relation of a logical entity does not in any way decide the question of its logical existence. If it is held that all logical entities without exception are in subject-relation to some "consciousness," it is still necessary to establish the distinction between "existing" and "non-existing" logical entities. This may be done by saying: a logical entity "exists" if it is "necessary," "of general validity"; whether such an attempt would prove successful or not is not our concern here; but it is our concern to insist that the mere relation of *all* logical entities to some consciousness is not capable of serving for a criterion to establish this distinction. *It is irrelevant to the problem of the existence of logical entities.*

If this distinction of the problems regarding the subject-relation of logical entities from those regarding the logical entities themselves is admitted, we may proceed to exemplify the latter types of problems. I shall call them *"Logic of Cognition," "Critique of Cognition,"* and *"Structure of Cognition."*

*"Logic of cognition"* treats of the relations of logical entities, not to a subject, but to each other. What are the propositions of plane geometry? Does a certain proposition $p_1$ "imply" another $p_2$? What consequences follow from certain assumptions? What laws are valid in the drawing of inferences? Logic of cognition constructs, from the true beginning, systems of cognition. Attempts such as Whitehead and Russell's "Principia Mathematica" are essentially examples of what is meant here by "logic of cognition." It is dogmatic in form; it does not justify or criticize; it exhibits, it hypothesizes, it proves; in brief, it constructs.

But with its special type of problems "logic of cognition," particularly in its beginning, combines (and often confuses) problems of the third type: *"critique of cognition."* "Critique" determines the logical "value" of systems of cognition; its main problem is the determination of the "truth" of a system, whilst "logic of cognition" should be indifferent to the question whether the hypotheses, whose consequences it develops, are true or not. When mathematicians exhibit sets of postulates of algebra, of geometry, they move in the realm of "logic of cognition"; when they add proofs of the "independence" or "consistency" of these postulates, they enter into the realm of "critique of cognition." Critique elaborates and applies certain criteria (which may be called "criteria of truth") to systems of cognition.

Construction and critical examination of systems of cognition, embracing as they are, leave still another type of problems dealing with the logical entities themselves. To state this new type of problems, I find it convenient to take up the old distinction between "form" and "content" and apply it to systems of cognition. Suppose we are studying the properties of parallelograms. We could write down a system of propositions, such as: the opposite sides are parallel; the opposite sides are equal; the opposite angles are equal; etc. But we might next ask: Are these propositions "independent" of each other? Or can we, in a plane geometry, by assuming some of them, deduce the others? We might then elaborate a different set of propositions, in which we proceed from some "defining" the parallelogram to others which we "prove." In both cases the same logical "content" is presented, namely, the properties of a parallelogram, but in different "form": a mere enumeration was "transformed" into a deductive system. And therewith a whole class of problems is presented all of which refer to the *structure of these possible forms,* in which the logical content of systems of cognition is or may be presented. What are the structural elements of a "deductive system"? How is it to be distinguished from "inductive systems"? What are the advantages of either form? What conditions must a certain logical content satisfy so that it can be put into the deductive system form? Are other, better forms existent or possible? What is the nature and function of "axioms," "definitions," "proofs"? The problem of the "new" in mathematics, the advantages and disadvantages of applying the deductive system form to "philosophy," all these are examples of the type of problems which constitute *"structure of cognition."*

These preliminary remarks may serve to direct attention away from some problems and toward the type to be examined in these studies, namely, the ones pertaining to "structure of cognition." Toward the necessity of keeping these problems distinct the following studies will add new evidence. Yet, the relation of these four disciplines to each other is so peculiarly close, that it is small wonder they have not been clearly distinguished before. No system can be presented without an appeal to the understanding of the reader, *i. e.,* without some subject-relation; every system will use concepts and propositions of logic of cognition; every system will have some structure, and endeavor to conform to the criteria of truth. This tends to confuse the issues; but if the emphasis is laid on the *problem* which is presented for solution, the distinction becomes simple. Every system *has* a definite structure, but this structure need not be the *problem* of every investigation; every system shall enter into a definite subject-relation, but this subject-relation need not be the problem of every investigation, etc.

Since the days of Kant, and largely in consequence of his work, our thinking has been controlled by the idea of "presupposition." Categories and fundamental judgments were to him "conditions of the possibility of experience," *i. e.,* that which is necessarily presupposed by experience itself. This idea has been extended to apply to sciences as a whole when we say: mathematics is presupposed by physics and attempts have been made to order the various disciplines in a series from this point of view of "presupposition." It is necessary to insist here, however, that this idea of "presupposition" leads readily to vagueness and confusion if applied promiscuously. It requires two restrictions.

In the first place it is by no means "self evident" that the various disciplines can really be arranged in serial order by this principle of presupposition; and this applies to the four disciplines which we have differentiated above. In a certain sense any one of them "presupposes" all the others. You can not study the subject-relation without using logical concepts and methods, without applying a definite ideal of truth, without putting the logical content of your study into a form of definite structure; and the same applies around the circle. Not only is it vain thus to try to find the "more fundamental" of the four, but it is positively misleading and injurious: we are apt to think that the questions of one can not be answered unless those of all the others are answered, somehow or other, first. We become so involved in "presuppositions" that we are unable to move a step forward or backward. The lesson of this predicament is instructive for all other cases. We must break through the idea of "presupposition" as applied to the various disciplines and recognize that each discipline makes its own presuppositions, its own hypotheses on which it builds; and in doing so may ignore the hypotheses of others. Mathematicians let "solids" interpenetrate each other, assume lines without breadth, weight, or color; to the physicist or psychologist such entities may be quite chimerical.

In the second place the idea of "presupposition" is meaningless, unless the "point of view" is added from which the presupposition is considered, in other words unless we state in the realm of which problem the particular presupposition is studied. The discipline which is "presupposed" by another in the realm of one problem may in turn presuppose it in the realm of a different problem. And thus we are led back again to the first distinction between the various *problems* which control our procedure: not methods, not objects, not principles and presuppositions separate these disciplines "psychology of cognition," "logic of cognition," "critique of cognition," "structure of cognition," but their problems!

If this is kept in mind, a paradox which may otherwise be puzzling will readily dissolve. In this study of the structure of systems we shall frequently "criticize" other accounts, and in this critique apply criteria which can be developed satisfactorily only in "critique of cognition." Thus we shall frequently apply the criterion of "completeness": certain accounts will be found defective in completeness in that they do not account for certain "facts." This would indeed be an infringement on the proper province of "critique of cognition," were it not for the circumstance that such critique is here merely incidental and for purposes of exposition. When some day the structure of systems will be studied more elaborately, we shall be able to dogmatically develop the various possible accounts, and then submit them to a systematic "critique." The growth of any science illustrates this, though what has been done more or less instinctively we can to-day see rationally. The change in procedure between Russell's "Principles of Mathematics" and Whitehead-Russell's "Principia Mathematica" is instructive in this respect, and illustrates the maxim that the reduction in polemic is proportional to the degree of logical perfection of a discipline; for the logical development of a system is one thing, its critical evaluation a second and distinct problem.

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## A SIMPLE METHOD FOR THE STUDY OF ENTOPTIC PHENOMENA

THE introspective examination of the eyè is interesting, both in experiments and in classroom work. The general name of entoptics for this subject was suggested by J. K. Listing. The method of studying the interior of one's own eyes, by letting light shine through a small pin-hole held close to the eye, has been carefully developed. Barrett constructed an elaborate instrument on this plan and made some detailed experiments which are reported in the *Proceedings of the Dublin Royal Society,* 1906. That the inside of the eye can be illuminated by the light reflected from a bright surface held close to the eyeball has been frequently mentioned, but the possible improvements in method that this fact provides have not been developed so far as I am aware, nor have their great advantages been appreciated.

A very simple and very effective apparatus for becoming acquainted with some of the characteristics and phenomena of one's own eye is provided by small silver beads strung on a wire in a spectacle frame. From the standpoint of psychology, perhaps the most important use of such an instrument is in the study of the movements of the iris.[1] If, for instance, three beads are strung for each eye on a wire adjusted to the spectacle frame so that they are horizontal just below or perpendicular to one side of the pupil, they will throw three circles of light upon different parts of the retina of each eye. For some experiments it is well to cover the frame with black cloth, allowing the beads to show through a slit. The beads may be moved back and forth and the intensity of the light increased or diminished by approaching it or removing from it until the middle circle is exactly tangent to the two others.

In the first place we have here a means of observing the reflex action of the pupils in both the eyes at the same time. Their coordination may be examined.

In the second place we have a means of measuring quite exactly

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[1] Badol reports an instrument to study dilations of the pupil in *Transactions de la Société de Biologie,* 1876. He used a cylinder and two cards with pin-holes. For the study of iris movements from the medical standpoint, see Bumke, "Pupillenstörungen," 1904, and Bache, "Pupillenlehre," 1908.